Lecture T3

Combined Mass and Energy Transients

We now consider processes in which the amounts of both mass and energy are changing in the system. In these cases, the material and energy balances are both differential equations, and it is not unusual for the complete mathematical description of the process to become quite complicated. In this lecture we restrict attention to relatively simple situations in which the material and energy balances decouple, so the material balance can be solved, independently of the energy balance. Then, the solution to the material balance is used in solving the energy balance. The few simple problems presented here will not make you an expert at solving transient problems, but they should serve to introduce you to the kinds of difficulties that can arise and how one addresses them.

T3.1 An Example with One Input and No Outputs

A 100-gal, enclosed, well-insulated, cylindrical tank initially holds 50-gal of water at 20°C. At the top of the tank is a small vent line, open to the atmosphere; see Figure T3.1. Near the bottom of the tank, a valved line is available to admit low-pressure steam to the tank. The steam is saturated at 3 bar. The objective is to heat the water to 70°C. The process starts when the valve in the steam line is opened, allowing steam to flow into the tank at a constant rate of 1 kg/min. The computational problems are to (a) determine the time required for the contents of the tank to reach 70°C and (b) determine the total mass of water in the tank when it reaches 70°C.

Let M be the total mass of water in the tank at any time t and let m_i be the flow rate of steam into the tank. Then the general material balance (T1.1) is

$$\frac{dM}{dt} = m_{\mathrm{i}} \tag{T3.1}$$

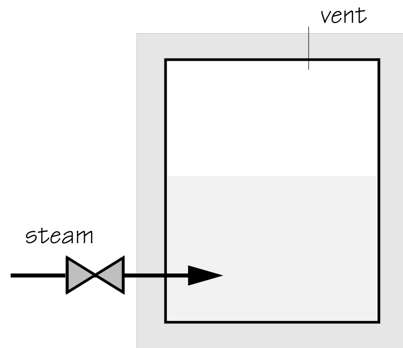


Figure T3.1 Schematic of an insulated tank having a single input for steam and no outputs. The losses through the vent are negligible over the time scale of interest.

Since the feed rate $m_i = 1$ kg/min is constant, (T3.1) can immediately be integrated to obtain the mass at any time t,

$$M(t) = M_o + m_i t \tag{T3.2}$$

where M_o is the initial mass of water in the tank. At 20°C, 50 gallons of water corresponds to $M_o = 189.6$ kg.

The general energy balance (T2.3) for this situation simplifies to

$$\frac{d(Mu)}{dt} = m_i h_i + W_b \tag{T3.3}$$

where u is the intensive internal energy of water in the tank, h_i is the intensive enthalpy of the steam, and $dW_b = -P\,d(Mv)$ is the boundary work done by the surface of the water compressing the vapor above it. Because of the vent, the pressure is constant on that surface; however, the lost of vapor to the surroundings is negligible, so we combine the PdV term with the internal energy. Then the lhs can be expressed in terms of the intensive enthalpy (h) for water in the tank,

$$\frac{d(Mh)}{dt} = m_i h_i \tag{T3.4}$$

The principal error is writing (T3.3) and (T3.4) occurs from neglect of the change in boundary energy (ΔE_b); that is, some enthalpy from the steam will be used to increase the temperature of the tank walls rather than increase the temperature of the water.

Our computational strategy is to solve the differential equation (T3.4) with the help of the material balance (T3.1) and (T3.2). We begin by expanding the lhs of (T3.4),

$$M\frac{dh}{dt} + h\frac{dM}{dt} = m_i h_i \tag{T3.5}$$

The two terms on the lhs mean that there are two contributions to the increase in enthalpy of the tank: changes in the thermodynamic state (dh/dt) and changes in the amount of material (dM/dt). We now use (T3.1) to eliminate dM/dt and collect terms,

$$M\frac{dh}{dt} = m_i(h_i - h) \tag{T3.6}$$

For the time dependence of $M(t)$, we substitute (T3.2),

$$(M_o + m_i t)\frac{dh}{dt} = m_i(h_i - h) \tag{T3.7}$$

This can be solved by separating variables,

$$\frac{dh}{(h_i - h)} = \frac{m_i dt}{(M_o + m_i t)} \tag{T3.8}$$

Let h_o be the intensive enthalpy of the water in the tank at $t = 0$, when steam is first admitted. Then integrating (T3.8) yields

$$-\ln\left(\frac{h_i - h}{h_i - h_o}\right) = \ln\left(\frac{M_o/m_i + t}{M_o/m_i}\right) \tag{T3.9}$$

or

$$\frac{h_i - h_o}{h_i - h} = 1 + \frac{m_i t}{M_o} \tag{T3.10}$$

This can be solved explicitly for h at any t or for t at any h; we choose the latter; hence,

$$t = \frac{M_o}{m_i}\left(\frac{h - h_o}{h_i - h}\right) \tag{T3.11}$$

Note that the elapsed time t is linear in the initial loading M_o, so if M_o is doubled, then it takes twice as long to reach the same final temperature. Further, t varies inversely with the steam feed rate min, so if we double the steam flow rate, then we halve the time needed to reach the same final temperature.

Steam tables give $h_o = 84$ kJ/kg for water at 20°C, $h_i = 2725$ kJ/kg for saturated steam at 3 bar, and $h = 293$ kJ/kg for water at the desired $T = 70$°C. Thus, (T3.11) becomes

$$t = \frac{189.6 \text{ kg}}{1 \text{ kg/min}}\left(\frac{293 - 84}{2725 - 293}\right) = 16.3 \text{ min} \tag{T3.12}$$

The steam must flow for 16.3 minutes to bring the contents of the tank to 70°C. At that point, the material balance (T3.2) gives

$$M = 189.6 \text{ kg} + (1 \text{ kg/min})16.3 \text{ min} = 206 \text{ kg} \tag{T3.13}$$

as the total amount of water in the tank. The complete heating curve for this situation is shown in Figure T3.2.

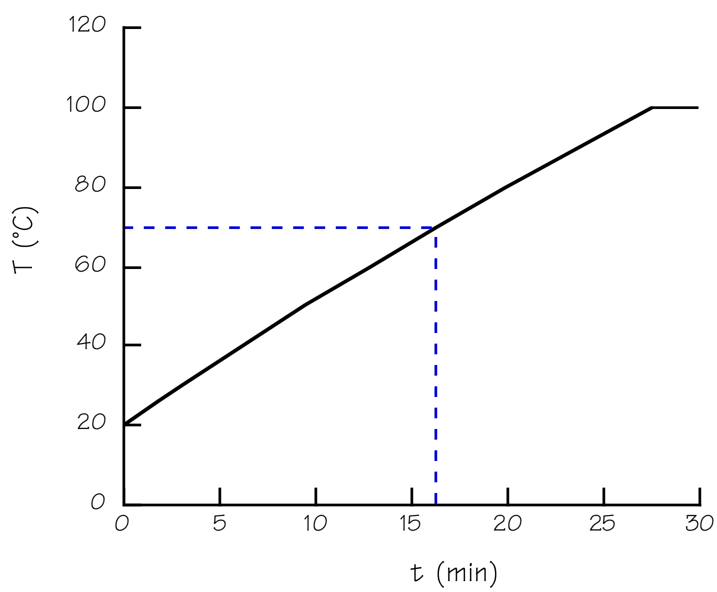


Figure T3.2 Heating curve for water in the tank shown in Figure T3.1. Curve computed from (T3.11) with enthalpies of saturated liquids from steam tables. After 16.3 min of flow, the water reaches 70°C. After 27.5 min, it reaches the normal boiling point; at that time, the tank holds 60 gallons of liquid water.

T3.2 An Example with Two Inputs and One Output

We now consider a more complicated version of the heating problem presented in the previous section. We again have a 100-gal, well-insulated, vented, cylindrical tank, initially holding 50 gallons of water. At the top of this tank, a line feeds water into the tank at a constant rate of 8 gal/min. From the bottom of the tank, a pump removes water at 8 gal/min; see Figure T3.3. Low-pressure steam (saturated at 3 bar) can be admitted into the tank, but initially the steam supply valve is closed, so the flow through the tank is initially a steady state. The feed water and tank contents are initially at 20°C.

The tank contents need to be heated so that water leaves the tank at 70°C. To achieve this, we will open the steam supply valve, allow steam to heat the tank contents to 70°C, then adjust the steam supply and outlet flow valves to a new steady state. These questions to be resolved are these:

(a) What mass flow rate of steam, m_s, is required to maintain the new steady state at 70°C?

(b) What value of m_s will we use during the transient that heats the water from 20°C to 70°C?

(c) How long will the transient heating last?

(d) When the new steady state is reached, what will be the total amount of water in the tank?

T3.2.1 Steady-State Solution

We take the entire contents of the tank as the system. When the new steady state is reached, the rate of water pumped from the tank, m_{so}, will be balanced by the mass flows of water, m_c, and steam, m_s, into the tank,

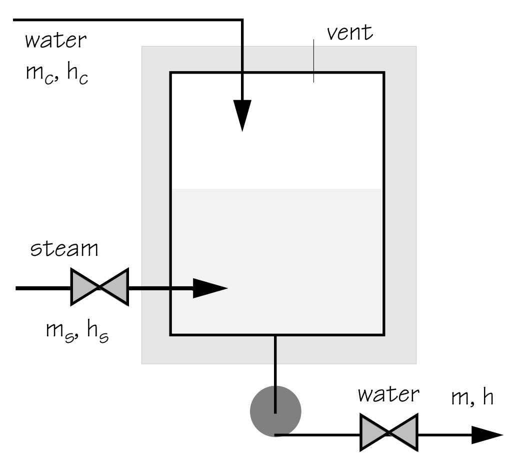


Figure T3.3 Schematic of an insulated tank provided with two inputs (water and steam) and one output (water).

$$m_{so} = m_c + m_s \tag{T3.14}$$

The steady-state process is adiabatic, workfree, with negligible changes in kinetic and potential energies, so the steady-state energy balance reduces to

$$\Delta H = 0 \tag{T3.15}$$

or

$$m_{so}h_{so} = m_c h_c + m_s h_s \tag{T3.16}$$

Using the material balance (T3.14) to eliminate m_{so} from the energy balance (T3.16), we can write

$$m_s = m_c\left(\frac{h_{so} - h_c}{h_s - h_{so}}\right) \tag{T3.17}$$

The constant feed rate of 8 gal/min corresponds to m_c = 30.3 kg/min. Steam tables give h_c = 84 kJ/kg, h_s = 2725 kJ/kg, and h_{so} = 293 kJ/kg for water at 70°C. Then (T3.17) becomes

$$m_s = 30.3\left(\frac{293 - 84}{2725 - 293}\right) = 2.6 \text{ kg/min} \tag{T3.18}$$

This is the feed rate of steam needed to maintain the tank contents at 70°C. Any flow rate less than this will not be sufficient to heat the feed water from 20°C to 70°C. This answers question (a) above.

However, if we use just the steady-state rate (T3.18) to heat the water during the transient, the water in the tank will only approach 70°C asymptotically. We prefer to use a higher steam flow rate than (T3.18), such as

$$m_s = 3.0 \text{ kg/min} \tag{T3.19}$$

When the tank contents reach 70°C, we will adjust the steam supply valve to maintain the steam flow at 2.6 kg/min and adjust the tank outlet valve to achieve the new steady state. The choice (T3.19) answers question (b).

T3.2.2 Transient Analysis

During the transient, cold water enters the tank at 8 gal/min (m_c = 30.3 kg/min) and water is pumped from the tank at the same rate, so $m = m_c$. In addition, steam is entering the tank at the value m_s selected in (T3.19). The general material balance for the transient is therefore

$$\frac{dM}{dt} = m_s \tag{T3.20}$$

where M is the amount of water in the tank at any time t. Integrating (T3.20) yields

$$M(t) = M_o + m_s t \tag{T3.21}$$

where M_o = 189.6 kg (50 gallons) is the amount of water in the tank at the start of the transient ($t = 0$).

For this situation, the general energy balance (T2.3) becomes

$$\frac{d(Mu)}{dt} = m_c h_c + m_s h_s - mh + W_b \tag{T3.22}$$

where h is the time-dependent enthalpy of water leaving the tank. In (T3.22), we treat the boundary work just as we did in § T3.1: we neglect the small amount of vapor lost through the vent, assume the expansion of the water's surface is isobaric, so $W_b = -P\,d(Mv)$, and combine this terms with the internal energy on the lhs. The result is

$$\frac{d(Mh)}{dt} = m_c h_c + m_s h_s - mh \tag{T3.23}$$

where we have used $m = m_c$. In writing (T3.23), we have assumed that the contents of the tank are well mixed, so the enthalpy (h) of the contents has the same value as the enthalpy of water pumped from the tank.

Note that the term $(m_c h_c + m_s h_s)$ is a constant during the transient; in fact, it is the steady-state enthalpy given by (T3.16), so we write (T3.23) as

$$\frac{d(Mh)}{dt} = m_{ss} h_{ss} - mh \tag{T3.24}$$

Expanding the lhs, we have

$$M\frac{dh}{dt} + h\frac{dM}{dt} = m_{ss} h_{ss} - mh \tag{T3.25}$$

Using the material balance (T3.20) for dM/dt and collecting terms, we find

$$M\frac{dh}{dt} = m_{ss}(h_{ss} - h) \tag{T3.26}$$

where we have used (T3.14) for (m_c + m_s). The time dependence of M is given by (T3.21); hence,

$$(M_o + m_s t)\frac{dh}{dt} = m_{ss}(h_{ss} - h) \tag{T3.27}$$

This differential equation is to be solved, at any time t, for the enthalpy h of water leaving the tank. It can be solved by separating variables,

$$\frac{dh}{(h_{ss} - h)} = \frac{m_{ss}dt}{(M_o + m_s t)} \tag{T3.28}$$

We integrate from $h = h_c$ at $t = 0$ to any subsequent time t, and find

$$\ln\left(\frac{h_{ss} - h}{h_{ss} - h_c}\right) = -\frac{m_{ss}}{m_s}\ln\left(\frac{M_o + m_s t}{M_o}\right) \tag{T3.29}$$

Algebraic rearrangement of (T3.29) allows us to express h explicitly as a function of t,

$$h = h_{ss} - \frac{h_{ss} - h_c}{(1 + (m_s t)/M_o)^{m_{ss}/m_s}} \tag{T3.30}$$

Note the limiting behavior: at $t = 0$, $h = h_c$, and as $t \to \infty$, $h \to h_{ss}$, which is the steady-state value. However, this would be a steady-state only with respect to energy. Unless valves were adjusted, there would still be a transient wrt mass; water would continue to accumulate in the tank until it overflowed through the vent. In other words, when h reaches h_{ss}, $dh/dt = 0$ in (T3.25), but dM/dt remains nonzero.

The numerical values for our situation are M_o = 189.6 kg, m_c = 30.3 kg/min, m_s = 3 kg/min with h_c = 84 kJ/kg, h_s = 2725 kJ/kg. Then (T3.14) gives m_{ss} = 33.3 kg/min and (T3.16) gives h_{ss} = 321.9 kJ/kg. At 70°C water has h = 293 kJ/kg, and solving (T3.30) for t to give this value of h, we find

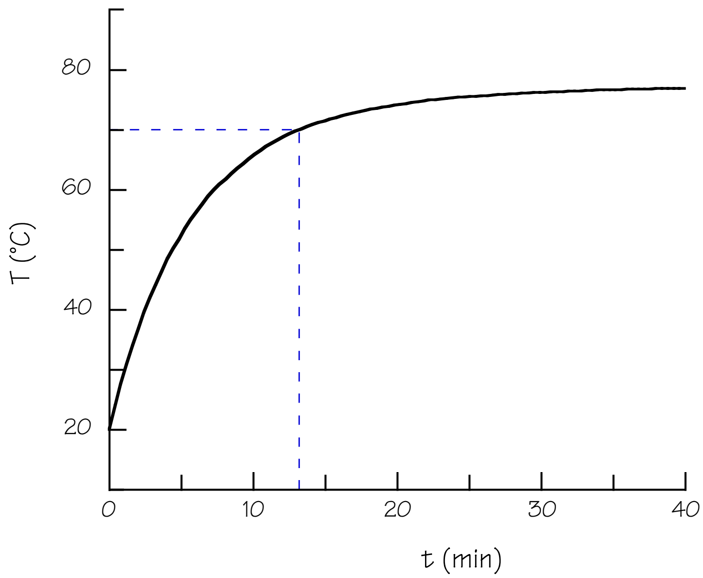


Figure T3.4 Heating curve for the water in the tank shown in Figure T3.3. After 13.2 min of steam flow, the water reaches the desired temperature 70°C. At that point, valves should be adjusted to achieve steady-state flow; if valve settings are not adjusted, then this curve shows that the water continues to heat until it reaches 76.9°C.

$$t = 13.2 \text{ min} \tag{T3.31}$$

This is the time required for the transient heating that increases the temperature of the water from 20°C to 70°C. At this point the material balance (T3.21) gives

$$M = 229 \text{ kg} \tag{T3.32}$$

as the total amount of water in the tank.

After 13.2 minutes of heating we would adjust the steam supply valve and water outlet valve to the new steady state. If we failed to make these valve adjustments, the water in the tank would continue to heat until it reached 76.9°C. At this point $h = h_{ss} = 321.9$ kJ/kg, and the enthalpy provided by the steam is just sufficient to bring the incoming cold water to 70°C. The complete heating curve is shown in Figure T3.4.

Problems

T3.1 During the heating process illustrated by the heating curve in Figure T3.2, the amount of water in the tank increases from 50 to 60 gallons over 27.5 min. Over this interval, estimate the flow rate of vapor through the vent. Is this flow negligible for the analysis performed in § T3.1?

T3.2 Consider the process described in § T3.1 for the situation shown in Figure T3.1, except let the transient start with the tank completely filled with liquid water. Thus, when the steam valve is opened, water flows out of the tank through the vent line. Assuming the mass flow is a steady state, and using all other conditions the same as in § T3.1, compute and plot the heating curve $T(t)$ for the contents of the tank.

T3.3 A one-quart thermos bottle contains 16 oz of liquid water, initially at 75°F. What volume of ice must be added to the water so that the combined contents reaches 45°F just as the last of the ice melts? At 1 atm the density of ice is about 0.92 g/cm^3 and the latent heat of melting is 1T3.5 Btu/lb$_m$. The process in this problem is a transient, yet the analysis differs from the example problems discussed in this lecture. Why?

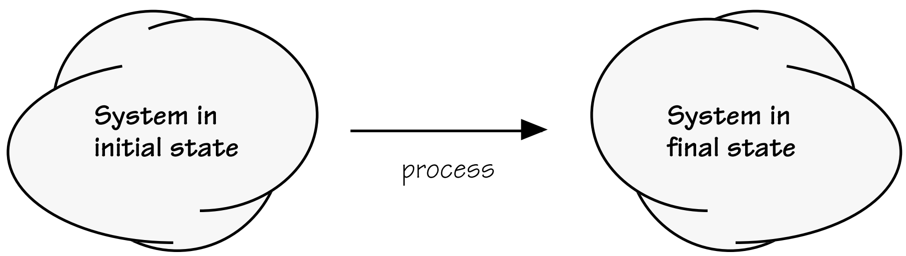
T3.4 Repeat the solution to the tank problem in § T3.1, but include the effects of heating the tank walls. The interior volume of the cylindrical tank is 100 gallons and it has an inside diameter of 2 ft. The wall thickness is 3/8-in. The tank is made of a low-alloy steel having a density of 3T2.6 lb$_m$/ft3 and a specific heat of 0.11 Btu/lb$_m$ F. All other conditions for the process are as described in § T3.1.

T3.5 A 100-gal vented, insulated, cylindrical tank initially holds 50 gallons of water at 20°C. The tank has two inputs: one for cold water (20°C), the other for warm water (80°C). (a) Determine the heating curve $T(t)$ for water in the tank when flows of both inputs are started simultaneously at 15 gpm. What is the temperature of water in the tank when water starts to overflow through the vent line? (b) Assume the mass flow achieves a steady state when overflow begins. Determine the new heating curve from the point of initial overflow until the energy balance achieves steady state.

T3.6 A rigid, insulated, 100-gal tank initially contains saturated steam at 10 bar. To reduce the pressure, the tank is fitted with a shower head for introducing water as a spray. Water is fed to the spray head at 20°C and a constant rate of 0.5 gpm. Under these conditions, how long must the spray be maintained to reduce the pressure to 1 bar? When the pressure reaches 1 bar, the spray is stopped; at that point, how much liquid water is in the tank and how much water vapor is present? Why is a spray used instead of simply pumping in a liquid stream? EXTRA CREDIT: Determine the pressure $P(t)$ and cooling curves $T(t)$ for the transient from 10 bar to 1 bar.

Summary for Part T

Transients are common industrial operations, for even the standard thermodynamic problem,



is accomplished via a transient. However, the real power of thermodynamics lies in its ability to identify energy requirements needed to perform a change even when the details of the change itself are unknown or unknowable. Most of the problems in this book are of this form.

We perform a transient analysis when we need to describe how quantities change with time. In most of those situations thermodynamics does not provide a sufficient characterization so that a complete determination of temporal behavior can be computed. Rates are opposed by resistances to driving forces, and resistances are not thermodynamic quantities. Nevertheless, in Lectures T1-T3, we have tried to convince you that thermodynamics is a necessary component of transient analyses.